

Look at the following equations:

$$x^2 - 5x - 6 = 0$$

$$(3x-2)^2 - 5(3x-2) - 6 = 0$$

$$\left(\frac{1}{2x}\right)^2 - 5\left(\frac{1}{2x}\right) - 6 = 0$$

$$x - 5\sqrt{x} - 6 = 0$$

$$x^4 - 5x^2 - 6 = 0$$

$$x^{-2} - 5x^{-1} - 6 = 0$$

Only the first equation is a quadratic equation. The others are "in the form (format)" of a quadratic.

- 1) At most 3 terms
- 2) The variable (or variable expression) occurs at most twice
- 3) The "smaller" variable squared = the "larger" variable

When these three are all true, we can solve the equation using any of the methods used to solve a quadratic equation:

- 1) Factoring
- 2) Completing The Square
- 3) Quadratic Formula

Ex1 Solve:  $(3x-2)^2 - 5(3x-2) - 6 = 0$

Factor  
 $[(3x-2) - 6] [(3x-2) + 1] = 0$

$$[(3x-2) - 6] = 0 \quad [(3x-2) + 1] = 0$$

$$3x - 8 = 0 \quad 3x - 1 = 0$$

$$3x = 8 \quad 3x = 1$$

$$x = \frac{8}{3} \text{ or } x = \frac{1}{3}$$

Ex2 Solve:  $(\frac{1}{2x})^2 - 5(\frac{1}{2x}) - 6 = 0$

Quadratic Formula:

$$a = 1 \quad b = -5 \quad c = -6$$

$$\frac{1}{2x} = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$$

$$\frac{1}{2x} = \frac{5 \pm \sqrt{25 + 24}}{2}$$

$$\frac{1}{2x} = \frac{5 \pm \sqrt{49}}{2}$$

$$\frac{1}{2x} = \frac{5+7}{2}$$

$$\frac{1}{2x} = \frac{5+7}{2}$$

$$\frac{1}{2x} = \frac{12}{2}$$

$$\frac{1}{2x} = 6$$

$$2x = \frac{1}{6}$$

$$x = \frac{1}{12}$$

or

$$\frac{1}{2x} = \frac{5-7}{2}$$

$$\frac{1}{2x} = \frac{-2}{2}$$

$$\frac{1}{2x} = -1$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

took  
reciprocal  
of both  
sides

Ex 3 Solve:  $3x + 4\sqrt{x} - 2 = 0$

$a = 3$   $b = 4$   $c = -2$

$$\sqrt{x} = \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)}$$

$$\sqrt{x} = \frac{-4 \pm \sqrt{16 + 24}}{6}$$

$$\sqrt{x} = \frac{-4 \pm \sqrt{40}}{6}$$

$$\sqrt{x} = \frac{-4 \pm 2\sqrt{10}}{6}$$

$$\sqrt{x} = \frac{\cancel{2}(-2 \pm \sqrt{10})}{\cancel{2} \cdot 3}$$

$$\sqrt{x} = \frac{-2 + \sqrt{10}}{3} ; \sqrt{x} = \frac{-2 - \sqrt{10}}{3}$$

$$\sqrt{x} = \frac{-2 + 3.2}{3} ; \sqrt{x} = \frac{-2 - 3.2}{3}$$

$$\sqrt{x} \approx 0.4 ; \sqrt{x} \approx \cancel{-1.7}$$

$$(\sqrt{x})^2 \approx (0.4)^2 ; (\sqrt{x})^2 \approx \cancel{(-1.7)^2}$$

$$x \approx 0.16$$

$$x \approx \cancel{2.89}$$

EX 4 Solve  $x^4 + 7x^2 - 18 = 0$

Factor:

$$(x^2 + 9)(x^2 - 2) = 0$$

$$x^2 + 9 = 0 \quad x^2 - 2 = 0$$

$$x^2 = -9 \quad x^2 = 2$$

$$\sqrt{x^2} = \sqrt{-9} \quad \sqrt{x^2} = \sqrt{2}$$

$$|x| = 3i$$

$$|x| = \sqrt{2}$$

$$x = \pm 3i \quad \text{or} \quad x = \pm \sqrt{2}$$