Theorem
A quadratic equation with roots r1 and r2 is

$$x^2 - (r_1 + r_2)x + r_1 r_2 = 0$$

or $a\left[x^2 - (r_1 + r_2)x + r_1 r_2\right] = 0$

Another way to write this is:

$$a\left[x^2 - (\text{sum of roots})x + (\text{product of roots})\right] = 0$$

Ex1 Find the equation whose roots are $\frac{2+i}{3}$ and $\frac{2-i}{3}$

Sum of roots $= \frac{2+i}{3} + \frac{2-i}{3} = \frac{4}{3}$

Product of roots $= \left(\frac{2+i}{3}\right)\left(\frac{2-i}{3}\right) = \frac{4-i^2}{9} = \frac{5}{9}$

Equation: $x^2 - \frac{4}{3}x + \frac{5}{9} = 0$

multiply by 9 to eliminate fractions

$$\frac{9}{1}\left[x^2 - \frac{4}{3}x + \frac{5}{9} = 0\right]$$

$$9x^2 - 12x + 5 = 0$$

Thm. If r_1 & r_2 are roots of $ax^2+bx+c=0$

$r_1+r_2 =$ sum of roots $= \frac{-b}{a}$

$r_1 \cdot r_2 =$ product of roots $= c/a$

Ex 2 Find the roots of $2x^2+9x+5=0$
Check your answers using the above theorem.

$$2x^2+9x+5=0$$

$$a=2 \quad b=9 \quad c=5$$

$$x = \frac{-9 \pm \sqrt{9^2-4(2)(5)}}{2(2)}$$

$$x = \frac{-9 \pm \sqrt{81-40}}{4}$$

$$r_1 = \frac{-9+\sqrt{41}}{4} \; ; \; r_2 = \frac{-9-\sqrt{41}}{4}$$

$$r_1+r_2 = \frac{-9+\sqrt{41}}{4} + \frac{-9-\sqrt{41}}{4} = \frac{-18}{4} = -\frac{9}{2} = \frac{-b}{a}$$

$$r_1 \cdot r_2 = \left(\frac{-9+\sqrt{41}}{4}\right)\left(\frac{-9-\sqrt{41}}{4}\right) = \frac{81-41}{16} = \frac{40}{16} = \frac{5}{2} = \frac{c}{a}$$

EX 3 Find a quadratic function
$f(x) = ax^2 + bx + c$ where the maximum is 8
and the graph has x-intercepts 3 and 7.

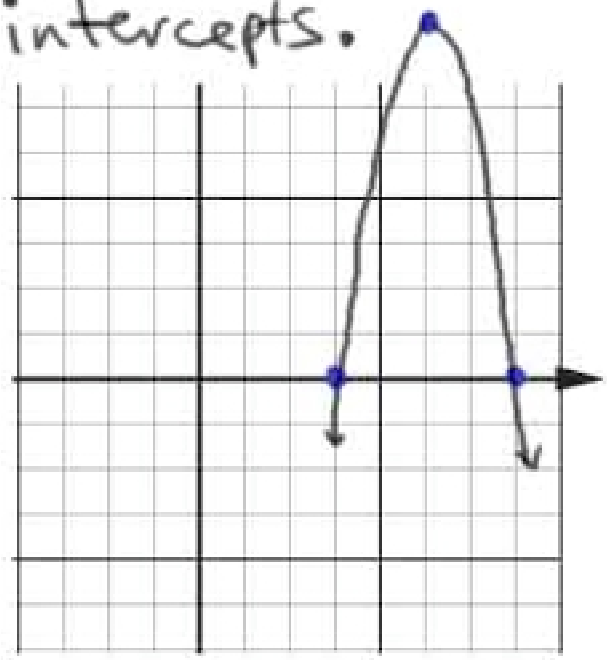
Remember: x-intercepts are zeros, roots, or
solutions of the equation.

sum: $3 + 7 = 10$
product: $3 \cdot 7 = 21$

$$f(x) = a(x^2 - 10x + 21)$$

maximum is 8, this is the y-value of the
vertex

the axis of symmetry is the midpoint of the x-intercepts, so to calculate the x-value of the vertex, find the average of the x-intercepts.



$$x = \frac{3+7}{2} = \frac{10}{2} = 5$$

vertex $(5, 8)$

$$f(x) = a(x^2 - 10x + 21)$$
$$8 = a(5^2 - 10(5) + 21)$$
$$8 = a(25 - 50 + 21)$$
$$8 = -4a$$
$$a = -2$$

$$f(x) = -2(x^2 - 10x + 21)$$
$$\boxed{f(x) = -2x^2 + 20x - 42}$$