8-5 The Remainder and Factor Theorems Page 372

Remainder Theorem

If $P(x)$ is a polynomial of degree n ($n>0$), then for any number r,
$P(x) = Q(x) \cdot (x-r) + P(r)$, where $Q(x)$ is a polynomial of degree n-1.
In other words: For the polynomial $P(x)$, the function value $P(r)$ is the remainder when $P(x)$ is divided by $x - r$.

To find $P(r)$, we can substitute "r" into the function $P(x)$ everywhere an x occurs and simplify or, use synthetic division by $(x-r)$ to find the remainder. Hence the name of the theorem, the Remainder Theorem.

Ex 1) Use synthetic division to find the value of $P(2)$ for the polynomial: $P(x) = 2x^4 + x^3 - 3x^2 + 5$

$0x$

$$
\begin{array}{r|rrrr|r}
2 & 2 & 1 & -3 & 0 & 5 \\
 & \downarrow & 4 & 10 & 14 & 28 \\
\hline
 & 2 & 5 & 7 & 14 & 33
\end{array}
$$

$$\boxed{P(2) = 33}$$

Factor Theorem

A polynomial P(x) has (x-r) as a factor if and only if r is a root of the equation P(x) = 0.

Ex 2) Determine if x + 1 is a factor of the polynomial:

$$P(x) = x^{12} - 3x^8 - 4x - 2$$

$$\begin{array}{r|rrrrrrrrrrrrr}
-1 & 1 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 & -4 & -2 \\
 & \downarrow & -1 & 1 & -1 & 1 & 2 & -2 & 2 & -2 & 2 & -2 & 2 & 2 \\
\hline
 & 1 & -1 & 1 & -1 & -2 & 2 & -2 & 2 & -2 & 2 & -2 & -2 & \boxed{0}
\end{array}$$

yes, the remainder is zero

(x+1) is a factor.

Ex3 Find the polynomial whose solutions are:

$1, -2, \frac{3}{2}$

$x=1, \ x=-2, \ x=\frac{3}{2}$

$(x-1)(x+2)\left(x-\frac{3}{2}\right)=0$

$(x-1)(x+2)\ 2\cdot\left(x-\frac{3}{2}\right)=0\cdot 2$

$\underbrace{(x-1)(x+2)}_{\text{Foil}}(2x-3)=0$

$(x^2+2x-x-2)(2x-3)=0$

$(x^2+x-2)(2x-3)=0$

$2x^3+2x^2-4x \qquad =0$

$\quad -3x^2-3x+6$

$2x^3-x^2-7x+6=0$

$\boxed{P(x)=2x^3-x^2-7x+6}$

Ex 4

Solve $x^3 + x + 10 = 0$ if -2 is a root.

$0x^2$

$$\begin{array}{r|rrrr|r} -2 & 1 & 0 & 1 & & 10 \\ & \downarrow & -2 & 4 & & -10 \\ \hline & 1x^2 & -2x & +5 & & 0 \end{array}$$

depressed equation

$x^2 - 2x + 5 = 0$ use Quadratic Formula

$a = 1 \quad b = -2 \quad c = 5$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 20}}{2}$$

$$x = \frac{2 \pm \sqrt{-16}}{2}$$

$$x = \frac{2 \pm 4i}{2}$$

$$x = \frac{\cancel{2}(1 \pm 2i)}{\cancel{2}}$$

$$x = 1 + 2i \; ; \; x = 1 - 2i$$