Rational Root Theorem

For $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

where all coefficients are integers.

Find a rational number c/d, where c and d are relatively prime. For c/d to be a root of P(x), c must be a factor of the constant and d must be a factor of the leading coefficient.

Ex 1) List all the possible rational roots of the polynomial:

$$P(x) = 3x^4 - 11x^3 + 10x - 4$$

c: $\pm 1 \quad \pm 2 \quad \pm 4$

d: $\pm 1 \quad \pm 3$

$\frac{c}{d}$: $\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm \frac{1}{3} \quad \pm \frac{2}{3} \quad \pm \frac{4}{3}$

Possible Rational Roots

Ex 2) Find all the roots of the polynomial:

$$P(x) = 3x^4 - 11x^3 + 10x - 4$$

From Ex 1, we know the possible rational roots are:

$$\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm \frac{1}{3} \quad \pm \frac{2}{3} \quad \pm \frac{4}{3}$$

12 possible rational roots

$$\begin{array}{r|rrrrr} 1 & 3 & -11 & 0 & 10 & -4 \\ & \downarrow & 3 & -8 & -8 & 2 \\ \hline & 3 & -8 & -8 & 2 & | -2 \end{array}$$

$\rightarrow$ Not zero, so 1 is Not a root

$$\begin{array}{r|rrrrr} -1 & 3 & -11 & 0 & 10 & -4 \\ & \downarrow & -3 & 14 & -14 & 4 \\ \hline & 3x^3 & -14x^2 & +14x & -4 & | \ 0 \end{array}$$

$\leftarrow$ zero

-1 is a root

(x+1) is a factor

Depressed Eq.

$$3x^3 - 14x^2 + 14x - 4$$

$\frac{c}{d}$: $\pm 1 \quad \pm 2 \quad \pm 4 \quad \pm \frac{1}{3} \quad \pm \frac{2}{3} \quad \pm \frac{4}{3}$

Keep trying the possible roots...

$$\begin{array}{c|cccc} \frac{2}{3} & 3 & -14 & 14 & -4 \\ & \downarrow & 2 & -8 & 4 \\ \hline & 3x^2 & -12x & +6 & \mid\ 0 \end{array}$$

zero $\frac{2}{3}$ is a root

Depressed Equation

$3x^2 - 12x + 6$

$a = 3 \quad b = -12 \quad c = 6$

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(6)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{144 - 72}}{6}$$

$$x = \frac{12 \pm \sqrt{72}}{6}$$

$$x = \frac{12 \pm 6\sqrt{2}}{6}$$

$$x = 2 \pm \sqrt{2}$$

All 4 Roots: $-1, \frac{2}{3}, 2+\sqrt{2}, 2-\sqrt{2}$