Rational Root Theorem

For \( P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \)
where all coefficients are integers.
Find a rational number \( c/d \), where \( c \) and \( d \) are relatively prime. For \( c/d \) to be a root of \( P(x) \), \( c \) must be a factor of the constant and \( d \) must be a factor of the leading coefficient.

Ex 1) List all the possible rational roots of the polynomial:

\[ P(x) = 3x^4 - 11x^3 + 10x - 4 \]

\[ \begin{align*}
    c: & \pm 1 \pm 2 \pm 4 \\
    d: & \pm 1 \pm 3 \\
    \frac{c}{d}: & \pm 1 \pm 2 \pm 4 \pm \frac{1}{3} \pm \frac{2}{3} \pm \frac{4}{3} \\
    \text{Possible Rational Roots} &
\end{align*} \]
Ex 2) Find all the roots of the polynomial:

\[ P(x) = 3x^4 - 11x^3 + 10x - 4 \]

From Ex 1, we know the possible rational roots are:

\[ \pm 1 \pm 2 \pm 4 \pm \frac{1}{3} \pm \frac{2}{3} \pm \frac{4}{3} \]

12 possible rational roots

\[ \begin{array}{cccc}
1 & 3 & -11 & 0 \\
1 & 3 & -8 & -8 & 2 \\
3 & -8 & -8 & 2 & 1 & -2
\end{array} \]

Not a root

\[ \begin{array}{cccc}
1 & 3 & -11 & 0 \\
1 & -3 & 14 & -14 & 4 \\
3 & -14 & 21 & -4 & 0
\end{array} \]

Zero

\(-1\) is a root

\( (x+1) \) is a factor

Depressed Eq.

\[ 3x^3 - 14x^2 + 14x - 4 \]

\[ \pm 1 \pm 2 \pm 4 \pm \frac{1}{3} \pm \frac{2}{3} \pm \frac{4}{3} \]

Keep trying the possible roots...
\[
\begin{array}{cccc}
3 & -14 & 14 & -4 \\
1 & 2 & -8 & 4 \\
\end{array}
\]

\[
3x^2 - 12x + 6 \quad \text{zero is a root}
\]

Depressed Equation

\[3x^2 - 12x + 6\]

\[a = 3 \quad b = -12 \quad c = 6\]

\[
x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(6)}}{2(3)}
\]

\[
x = \frac{12 \pm \sqrt{144 - 72}}{6}
\]

\[
x = \frac{12 \pm \sqrt{72}}{6}
\]

\[
x = \frac{12 \pm 6\sqrt{2}}{6}
\]

\[x = 2 \pm \sqrt{2}\]

All 4 Roots: 
-1, \frac{2}{3}, 2+\sqrt{2}, 2-\sqrt{2}