

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

- 1) A & B have the same sign (both positive or both negative)
- 2) A & B are different numbers (if they were the same, this would be a circle).

Ex 1) For the ellipse find the center, vertices, and foci.

$$4x^2 + 9y^2 - 16x + 18y - 11 = 0$$

$$4x^2 - 16x + 9y^2 + 18y = 11$$

$$4(x^2 - 4x) + 9(y^2 + 2y) = 11$$

$$4(x^2 - 4x + 4) + 9(y^2 + 2y + 1) = 11 + 16 + 9$$

$$\frac{4(x-2)^2}{36} + \frac{9(y+1)^2}{36} = \frac{36}{36}$$

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{4} = 1$$

horizontal center $(2, -1)$

$$a = 3 \leftrightarrow b = 2 \downarrow$$

vertices: $(-1, -1)$ $(5, -1)$
 $(2, 1)$ $(2, -3)$

$$c = \sqrt{9-4} = \sqrt{5} \doteq 2.2 \leftrightarrow$$

foci: $(-0.2, -1)$ $(4.2, -1)$

Ex 2) For the ellipse find the center, vertices, and the foci.

$$9x^2 + 4y^2 + 54x - 8y + 49 = 0$$

$$9x^2 + 54x + 4y^2 - 8y = -49$$
$$9(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = -49$$

$+81$
 $+4$

$$\frac{9(x+3)^2}{36} + \frac{4(y-1)^2}{36} = \frac{36}{36}$$

$$\boxed{\frac{(x+3)^2}{4} + \frac{(y-1)^2}{9} = 1}$$

vertical center $(-3, 1)$

$$a = 3 \updownarrow \quad b = 2 \leftrightarrow$$

vertices: $(-3, 4)$ $(-3, -2)$

$(-5, 1)$ $(-1, 1)$

$$c = \sqrt{9-4} = \sqrt{5} \doteq 2.2 \updownarrow$$

foci: $(-3, 3.2)$ $(-3, -1.2)$