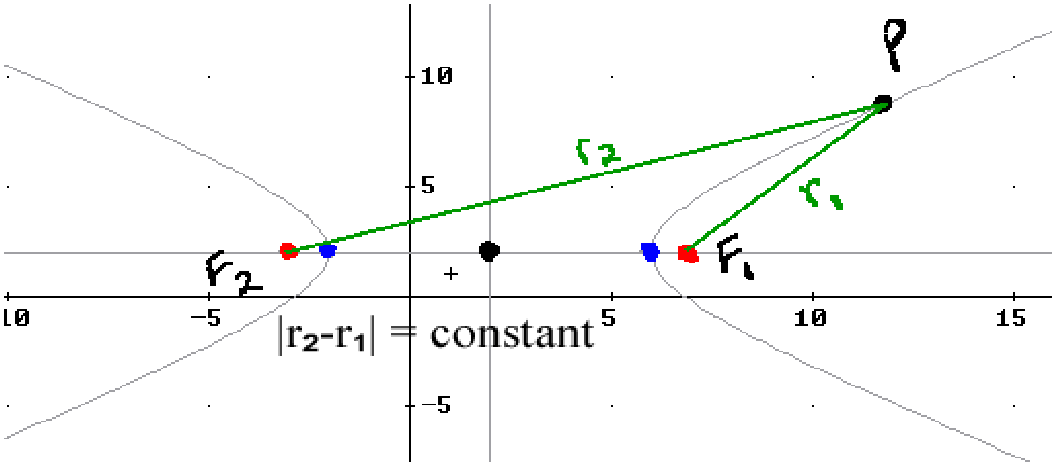
9-5 Part 1 Hyperbolas in Standard Form Page 426

Alg 2 Standard 16.0: Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it.

**Geometry Reminder:**

A **hyperbola** is the set of all points P in a plane such that the absolute value of the difference of the distances from P to two fixed points $F_1$ and $F_2$ is constant. The fixed points are the **foci** of the hyperbola. The line segments $r_1$ and $r_2$ are called the **focal radii**.

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Ex 1) For the hyperbola find the center, vertices, foc eccentricity, draw the asymptotes and then graph.

$$\frac{(x-2)^2}{1} - \frac{(y-5)^2}{9} = 1$$

horizontal center $(2,5)$

$a = 1 \leftrightarrow$ $b = 3 \updownarrow$

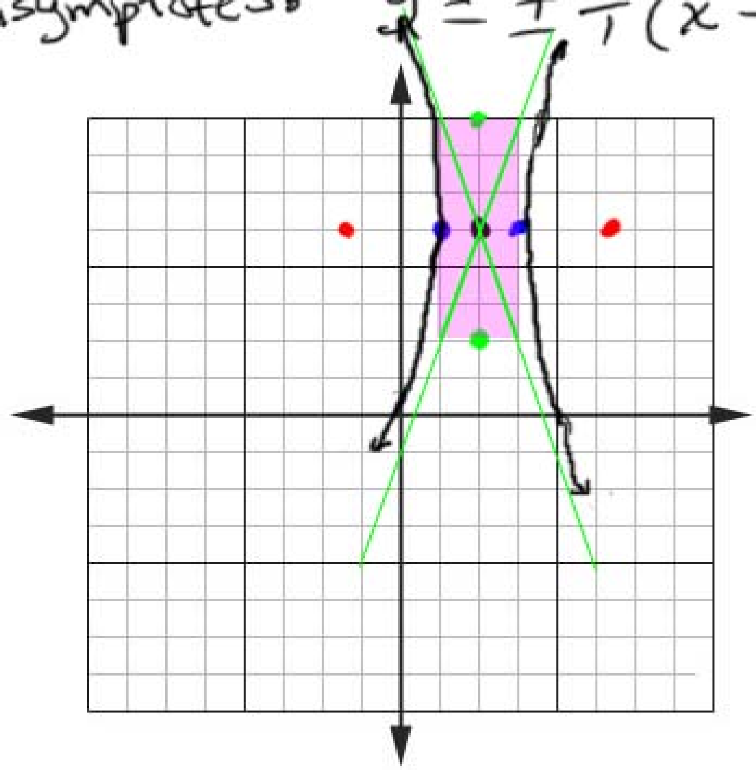
vertices: $(1,5)$ $(3,5)$

$c = \sqrt{1+9} = \sqrt{10} \doteq 3.2 \leftrightarrow$

foci: $(-1.2,5)$ $(5.2,5)$

eccentricity $e = \frac{c}{a}$ $e = \frac{\sqrt{10}}{1}$

asymptotes: $y = \pm\frac{3}{1}(x-2)+5$



$y = 3(x-2)+5$

$y = 3x - 6 + 5$

$y = 3x - 1$

$y = -3(x-2)+5$

$y = -3x + 6 + 5$

$y = -3x + 11$

Ex 2) For the hyperbola find the center, vertices, foci, eccentricity, draw the asymptotes, and then graph.

$$\frac{(y+3)^2}{25} - \frac{(x+1)^2}{16} = 1$$ vertical center$(-1, -3)$

$a = 5 \updownarrow$ $b = 4 \leftrightarrow$

vertices: $(-1, 2)$ $(-1, -8)$

$c = \sqrt{25+16} = \sqrt{41} \doteq 6.4 \updownarrow$

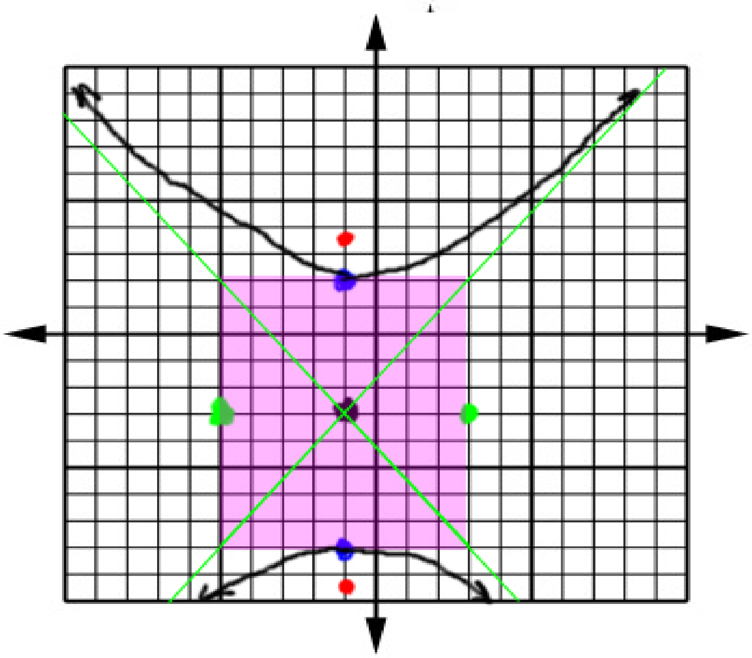
foci: $(-1, 3.4)$ $(-1, -9.4)$

eccentricity: $e = \frac{c}{a}$ $e = \frac{\sqrt{41}}{5}$

asymptotes: $y = \pm\frac{5}{4}(x + +1) + -3$

$y = \frac{5}{4}(x+1) - 3$ | $y = -\frac{5}{4}(x+1) - 3$

$\boxed{y = \frac{5}{4}x - 1\frac{3}{4}}$ | $\boxed{y = -\frac{5}{4}x - 4\frac{1}{4}}$



The eccentricity of an hyperbola is: $e > 1$.

Below is a table of eccentricities for the conic sections:

| Conic Section | Interval | The ratio e |
|---|---|---|
| Circle | e = 0 | 0 |
| Ellipse | $0 < e < 1$ | $\frac{c}{a}$ |
| Parabola | e = 1 | 1 |
| Hyperbola | e > 1 | $\frac{c}{a}$ |

Theorem: For a hyperbola of the form:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

where $c = \sqrt{a^2 + b^2}$

The eccentricity $e = \frac{c}{a}$