

9-5 Part 2 Hyperbolas in General Form

$$Ax^2 + By^2 + Cx + Dy + E = 0$$

- 1) A & B must have different signs (one positive and the other negative).
- 2) A & B must be different numbers or opposites (same number with different signs)

Ex 1) For the hyperbola find the center, vertices, and foci.

$$x^2 - y^2 - 2x - 4y - 4 = 0$$

$$\begin{aligned} x^2 - 2x &\quad - y^2 - 4y = 4 \\ (x^2 - 2x + 1) - \cancel{(y^2 + 4y + 4)} &= 4 \\ \frac{(x-1)^2}{1} - \frac{(y+2)^2}{4} &= 1 \end{aligned}$$

horizontal center $(1, -2)$

$$a = 1 \leftrightarrow b = 1$$

vertices: $(0, -2)$ $(2, -2)$

$$c = \sqrt{1+1} = \sqrt{2} \doteq 1.4 \leftrightarrow$$

$$\text{foci: } (-0.4, -2) \quad (2.4, -2)$$

Ex 2) For the hyperbola find the center, vertices, and foci.

$$-9x^2 + 4y^2 - 54x - 8y - 113 = 0$$

$$-9x^2 - 54x + 4y^2 - 8y = 113$$

$$-9(x^2 + 6x + 9) + 4(y^2 - 2y + 1) = 113$$

-81
+4

$$-\frac{9(x+3)^2}{36} + \frac{4(y-1)^2}{36} = \frac{36}{36}$$

$$-\frac{(x+3)^2}{4} + \frac{(y-1)^2}{9} = 1$$

$$\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$$

vertical center $(-3, 1)$

$$a = 3 \uparrow \quad b = 2 \leftrightarrow$$

$$\text{vertices: } (-3, 4) \quad (-3, -2)$$

$$c = \sqrt{9+4} = \sqrt{13} \approx 3.6 \downarrow$$

$$\text{foci: } (-3, 4.6) \quad (-3, -2.6)$$