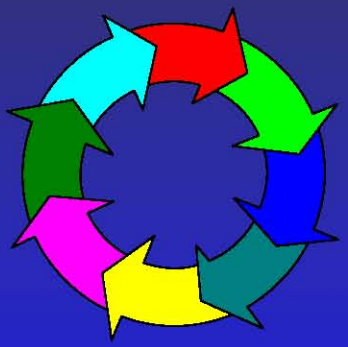
Chapter 12: Recursion



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Recursion: Definitions

- Recursion
  - The process of a subprogram (function) calling itself.
    - A clearly defined *stopping state* must exist.
      - The well defined termination of the recursive process.
    - Any recursive subprogram could be rewritten using iteration (for, while, do…while)
  - Recursive step
    - A step in the recursive process that solves a similar problem of a smaller size and will eventually lead to a termination of the process.

Recursive Problems in Mathematics

- Factorials
  - The definition of factorials is as follows:
    - 0! = 1; 1! = 1; for n>1, n! = n*(n-1)*(n-2)*…2*1
    - To find n! you can calculate n*(n-1)!
    - Each step solves a similar problem of smaller size.
- Fibonacci sequence
  - The first term is 1, the second term is also 1, and every successive term is the sum of the previous two terms.
- An Arithmetic Series using sigma notation $\sum_{x=1}^{n} x$

Recursive Example 1

- First recursion example
  - (do not compile or run, there is no stopping state)

recursion1.cpp recursion1.txt

```
void Example (int value)
{
   cout<<"Hello There! "<<value<<endl;
   Example (value + 1);
}
```

Recursive Example 2

- Second Example

recursion2.cpp recursion2.txt

```
void Example2 (int value)
{
   cout<<"Hello, value= "<<value<<endl;
   if (value < 5)
     Example2 (value + 1);
}
```

Example 2 Animation: 1

Value

1

Monitor

Hello, value = 1

Value < 5

TRUE

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

Example 2 Animation: 2

Value

2

Monitor

Hello, value = 1

Hello, value = 2

Value < 5

TRUE

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

Example 2 Animation: 3

Value

3

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Value < 5

TRUE

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

Example 2 Animation: 4

Value

4

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Value < 5

TRUE

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

Example 2 Animation: 5

Value

5

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Value < 5

FALSE

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
```

Example 2 Animation: 6

Value

5

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Value < 5

FALSE

Tail-recursive Algorithms

- **Tail-recursive** algorithms perform no work after the recursive call.
  - Examples 1 and 2 are tail recursive as well as the following example:

```
int tailRecursiveFactorial (int n, int result){
   if (n == 1)
      return result;
   else
      return tailRecursiveFactorial (n - 1, n * result);
}
```

Recursive Example 3

- Third Example (not tail recursive)

recursion3.cpp recursion3.txt

```
void Example3 (int value)
{
  cout<<"Hello, value= "<<value<<endl;
  if (value < 5)
     Example3 (value + 1);
  cout<<"Goodbye, value= "<<value<<endl;
}
```

Example 3

Value

1

Monitor

Hello, value = 1

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

2

Monitor

Hello, value = 1

Hello, value = 2

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

3

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

4

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

5

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

5

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

```
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

4

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

Goodbye, value = 4

```
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

3

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

Goodbye, value = 4

Goodbye, value = 3

```
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

2

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

Goodbye, value = 4

Goodbye, value = 3

Goodbye, value = 2

```
cout<<"Goodbye, value= "<<value<<endl;
```

```
cout<<"Hello, value= "<<value<<endl;
 if (value < 5)
   Example2 (value + 1);
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

1

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

Goodbye, value = 4

Goodbye, value = 3

Goodbye, value = 2

Goodbye, value = 1

```
cout<<"Goodbye, value= "<<value<<endl;
```

Example 3

Value

1

Monitor

Hello, value = 1

Hello, value = 2

Hello, value = 3

Hello, value = 4

Hello, value = 5

Goodbye, value = 5

Goodbye, value = 4

Goodbye, value = 3

Goodbye, value = 2

Goodbye, value = 1

Recursive Example 4

- Arithmetic Series (sigma) Example $\sum_{x=1}^{n} x$

P668ex1.cpp
P668ex1.txt

```
int sigma(int n)
{
    if(n <= 1)
        return n;
    else
        return n + sigma(n-1);
}
```

Sigma Example Function Calls

- If we call the sigma function with the statement, sum=sigma(5)
- the else portion of the first function calls the function again, return 5 + sigma(4);
  - at this point the value of sigma (4) must be computed
  - this calls return 4 + sigma (3)
  - which calls return 3 + sigma(2)
  - then return 2 + sigma(1)
  - finally this returns the value of 1.

Visualizing the sigma Function Calls

- return 5 + sigma(4)
  - return 4 + sigma(3)
    - return 3 + sigma(2)
      - return 2 + sigma(1)
        - return 1
      - at the end of the recursive calls the steps are reversed for assigning the values.
        - return 1
      - return(2+1) (=3)
    - return(3+3) (=6)
  - return(4+6) (=10)
- return(5+10) (=15)

Stacks

- Imagine a stack as a pile of trays in a cafeteria. The last tray put on the stack is the first one taken off of the stack.
  - This is what occurs in memory when a subprogram calls itself.
  - A stack is a dynamic data structure where access can be made only from one end. This is called a Last In First Out (LIFO) structure.

Stack Animation

- Level 5 n: 1 Sigma: 1
- Level 4: n: 2 Sigma: 3
- Level 3: n: 3 Sigma: 6
- Level 2: n: 4 Sigma: 10
- Level 1: n: 5 Sigma: 15

Reverse Print Recursion Example

- This example will read a character of text from a file until a period is read. It then uses recursive techniques to print the text in reverse order.

Revprint.cpp Revprint.txt

- This is a short sentence.
- .ecnetnes trohs a si sihT

The Costs and Benefits of Recursion

- Costs
  - A recursive process that requires N recursive calls uses N+1 units of stack memory and processor time to manage the process.
  - An equivalent iterative process always uses one stack of memory and processor time to manage the function call, therefore, recursion requires more memory and processor power than the non-recursive process.
- Benefits
  - Short, simple & elegant solutions using divide & conquer algorithms (solve the problem by repeatedly dividing it into simpler problems.)

Some Recursive Algorithms

- The Binary Search
- The Quick Sort
- The Merge Sort

M. C. Escher & Recursion

- M.C. Escher "Drawing Hands" lithograph
- Illustrates the concept of Recursion